

## Computational PDEs

1.1 Course Number: MA 321

1.2 Contact Hours: 41 (L)

Credits: 9 [LTP: 3-0-0]

1.3 Semester-offered: Odd

1.4 Prerequisite: Linear Algebra; Calculus; Differential Equations; Numerical Methods and Matlab

1.5 Syllabus Committee Member: Dr. C. Kundu, Dr. M.K. Rajpoot (convener), Dr. A. Kumar, Dr. G. Rakshit.

### 2. Objective:

- Students should learn the principles for designing numerical schemes for PDEs.
- Students should learn to make a connection between the mathematical equations or properties and the corresponding physical meanings.
- Students should be able to analyze the consistency, stability, and convergence of a numerical scheme.
- Students should know, for each type of PDEs (hyperbolic, parabolic, and elliptic), what kind of numerical methods are best suited for and the reasons behind these choices.
- Students should be able to use a programming language or software (Matlab or Maple) to implement and test the numerical schemes.

### 3. Course Content:

Unit-wise distribution of content and number of lectures

Unit	Topics	Sub-topic	Lectures
1	<b>Parabolic partial differential equations</b>	Explicit and implicit finite difference schemes; Truncation errors and consistency; Stability analysis: matrix method, maximum principle, Fourier analysis, energy method; maximum principle; Thomas algorithm; ADI method; general boundary conditions and nonlinear problems.	11
2	<b>Stability Criterion</b>	Consistency; convergence and stability; mesh and norms; finite difference approximations, consistency; order of accuracy and convergence; stability and the Lax equivalence theorem; calculating stability conditions.	10
3	<b>Hyperbolic partial differential</b>	Characteristics, The CFL condition, error analysis of the upwind schemes, Fourier analysis of the upwind scheme, The Lax-Wendroff scheme, The Lax-	10

	<b>equations</b>	Wendroff method for conservation laws.	
4	<b>Elliptic partial differential equations</b>	A model problem; Error analysis of the model problem; The general diffusion equation; boundary conditions on a curved boundary; Error analysis using a maximum principle; Asymptotic error estimates.	10
		<b>Total</b>	41

#### 4. Readings

##### 4.1 Textbook:

- I. K. W. Morton and D. F. Mayers, *Numerical Solution of Partial Differential Equations*, Cambridge University Press, 2<sup>nd</sup> Ed.
- II. W. F. Ames, *Numerical Methods for Partial Differential Equations*, Academic Press, 3<sup>rd</sup> Ed.

##### 4.2 Reference books:

- I. Courant, R. and Hilbert, D., *Methods of Mathematical Physics, Vol-2: Partial Differential Equations*, New York, Wiley-Interscience.

#### 5 Outcome of the Course:

After completion of the course students

- Should be able to apply numerical methods for partial differential equations.
- Have ability to determine the stability/convergence criterion for a numerical scheme.
- Would be able to apply the methods to solve physical/engineering problems.